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Explaining Pure Spinor Superspace

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In the pure spinor formalism for the superstring and supermembrane, supersymmetric invariants are constructed by integrating over five θ's in d=10 and over nine θ's in d=11. This pure spinor superspace is easily explained using the superform (or "ectoplasm") method developed by Gates and collaborators, and generalizes the standard chiral superspace in d=4. The ectoplasm method is also useful for constructing d=10 and d=11 supersymmetric invariants in curved supergravity backgrounds.

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1. Introduction

The conventional method for constructing supersymmetric invariants is to integrate superfields over a superspace which contains both x^m and θ^α variables. The number of θ's which must be integrated depends both on the spacetime dimension and on the constraints satisfied by the superfields. For example, d=4 supersymmetric invariants can be constructed either by using real superfields and integrating over four θ's, or by using chiral superfields and integrating over two θ's. Although it is somewhat non-trivial to generalize these d=4 supersymmetric invariants in a curved supergravity background, this can be done by inserting the appropriate constrained supervielbeins into the superspace integral.

This conventional method for constructing supersymmetric invariants is less useful in higher spacetime dimensions which involve more θ's. For example, the construction of d=10 super-Poincaré invariant expressions using unconstrained superfields would require integration over 16 θ's, which means that the supersymmetric invariants typically involve terms with eight spacetime derivatives. Although one can try to define constrained d=10 superfields which allow integration over fewer than 16 θ's, finding an appropriate set of constraints is not easy. Furthermore, if one finds a suitable set of constraints, it is not obvious how to generalize them in a curved supergravity background.

Over the last six years, an appropriate set of constraints for d=10 and d=11 superfields has been discovered using the pure spinor formalism for the superstring and supermembrane [1][2]. Using the constraints coming from these pure spinor formalisms, d=10 and d=11 supersymmetric invariants in a flat background have been constructed involving as few as two spacetime derivatives. These supersymmetric invariants naturally arise as on-shell scattering amplitudes in the pure spinor approach.

For example, N=1 d=10 supersymmetric invariants can be constructed from a superfield $f_{\alpha_1\alpha_2\alpha_3}(x,\theta)$ which satisfies the constraint

$$\lambda^\beta \lambda^{\alpha_1}\lambda^{\alpha_2}\lambda^{\alpha_3} D_\beta f_{\alpha_1\alpha_2\alpha_3}(x,\theta) = 0 \tag{1.1}$$

where $\alpha = 1$ to 16 is a d=10 spinor index, $D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \gamma^m_{\alpha\beta}\theta^\beta$ is the superspace derivative, and λ^α is a bosonic spinor satisfying the pure spinor condition that $\lambda\gamma^m\lambda = 0$ for $m = 0$ to 9. The N=1 d=10 supersymmetric invariant is then obtained by integrating over five of the 16 θ's as

$$T^{((\alpha_1\alpha_2\alpha_3))[\delta_1\ldots\delta_5]} \int d^{10}x \int (d^5\theta)_{\delta_1\ldots\delta_5}\ f_{\alpha_1\alpha_2\alpha_3}(x,\theta) \tag{1.2}$$

where

$$T^{\alpha_1\alpha_2\alpha_3\ \delta_1\ldots\delta_5} = \gamma_m^{\alpha_1\delta_1}\gamma_n^{\alpha_2\delta_2}\gamma_p^{\alpha_3\delta_3}(\gamma^{mnp})^{\delta_4\delta_5} \tag{1.3}$$

and $T^{((\alpha_1\alpha_2\alpha_3))[\delta_1\ldots\delta_5]}$ is obtained from (1.3) by antisymmetrizing in the δ indices, symmetrizing in the α indices, and subtracting γ-matrix trace terms in the α indices so that $\gamma^m_{\alpha_1\alpha_2}T^{((\alpha_1\alpha_2\alpha_3))[\delta_1\ldots\delta_5]} = 0$. For example, the cubic d=10 super-Yang-Mills coupling is given by (1.2) where $f_{\alpha_1\alpha_2\alpha_3} = A_{\alpha_1}A_{\alpha_2}A_{\alpha_3}$ and $A_\alpha(x,\theta)$ is the on-shell super-Yang-Mills spinor gauge superfield [1].

One can also construct N=2 d=10 supersymmetric invariants from a superfield $f_{\alpha_1\alpha_2\alpha_3\widehat{\beta}_1\widehat{\beta}_2\widehat{\beta}_3}(x,\theta,\widehat{\theta})$ which satisfies the constraints

$$\lambda^\gamma\lambda^{\alpha_1}\lambda^{\alpha_2}\lambda^{\alpha_3}\widehat{\lambda}^{\widehat{\beta}_1}\widehat{\lambda}^{\widehat{\beta}_2}\widehat{\lambda}^{\widehat{\beta}_3}D_\gamma f_{\alpha_1\alpha_2\alpha_3\widehat{\beta}_1\widehat{\beta}_2\widehat{\beta}_3}(x,\theta,\widehat{\theta}) = 0, \tag{1.4}$$

$$\widehat{\lambda}^{\widehat{\gamma}}\widehat{\lambda}^{\widehat{\beta}_1}\widehat{\lambda}^{\widehat{\beta}_2}\widehat{\lambda}^{\widehat{\beta}_3}\lambda^{\alpha_1}\lambda^{\alpha_2}\lambda^{\alpha_3}D_{\widehat{\gamma}} f_{\alpha_1\alpha_2\alpha_3\widehat{\beta}_1\widehat{\beta}_2\widehat{\beta}_3}(x,\theta,\widehat{\theta}) = 0,$$

where $\alpha = 1$ to 16 and $\widehat{\beta} = 1$ to 16 are d=10 spinor indices which are either of opposite chirality (for N=2A) or of the same chirality (for N=2B), $D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \gamma^m_{\alpha\beta}\theta^\beta$ and $D_{\widehat{\alpha}} = \frac{\partial}{\partial\widehat{\theta}^{\widehat{\alpha}}} + \gamma^m_{\widehat{\alpha}\widehat{\beta}}\widehat{\theta}^{\widehat{\beta}}$ are the N=2 superspace derivatives, and λ^α and $\widehat{\lambda}^{\widehat{\beta}}$ are bosonic spinors satisfying the pure spinor conditions that $\lambda\gamma^m\lambda = \widehat{\lambda}\gamma^m\widehat{\lambda} = 0$. The N=2 d=10 supersymmetric invariant is then obtained by integrating over five θ's and five $\widehat{\theta}$'s as

$$T^{((\alpha_1\alpha_2\alpha_3))[\delta_1\ldots\delta_5]}T^{((\widehat{\beta}_1\widehat{\beta}_2\widehat{\beta}_3))[\widehat{\gamma}_1\ldots\widehat{\gamma}_5]}\int d^{10}x\int(d^5\theta)_{\delta_1\ldots\delta_5}\int(d^5\widehat{\theta})_{\widehat{\gamma}_1\ldots\widehat{\gamma}_5}\ f_{\alpha_1\alpha_2\alpha_3\widehat{\beta}_1\widehat{\beta}_2\widehat{\beta}_3}. \tag{1.5}$$

Finally, d=11 supersymmetric invariants can be constructed from a superfield $f_{\underline{\alpha}_1\ldots\underline{\alpha}_7}(x,\theta)$ which satisfies the constraint

$$\lambda^{\underline{\gamma}}\lambda^{\underline{\alpha}_1}\ldots\lambda^{\underline{\alpha}_7}D_{\underline{\gamma}}f_{\underline{\alpha}_1\ldots\underline{\alpha}_7}(x,\theta) = 0 \tag{1.6}$$

where $\underline{\alpha} = 1$ to 32 is a d=11 spinor index, $D_{\underline{\alpha}} = \frac{\partial}{\partial\theta^{\underline{\alpha}}} + \gamma^{\underline{m}}_{\underline{\alpha}\underline{\beta}}\theta^{\underline{\beta}}$ is the d=11 superspace derivative, and $\lambda^{\underline{\alpha}}$ is a bosonic spinor satisfying the condition that $\lambda\gamma^{\underline{m}}\lambda = 0$ for $\underline{m} = 0$ to 10. The d=11 supersymmetric invariant is obtained by integrating over 9 of the 32 θ's as

$$T^{((\underline{\alpha}_1\ldots\underline{\alpha}_7))[\underline{\delta}_1\ldots\underline{\delta}_9]}\int d^{11}x\int(d^9\theta)_{\underline{\delta}_1\ldots\underline{\delta}_9}\ f_{\underline{\alpha}_1\ldots\underline{\alpha}_7}(x,\theta). \tag{1.7}$$

As in d=10, $T^{((\underline{\alpha}_1\ldots\underline{\alpha}_7))[\underline{\delta}_1\ldots\underline{\delta}_9]}$ is a Lorentz-invariant tensor which is antisymmetric in the $\underline{\delta}$ indices and symmetric γ-matrix traceless in the $\underline{\alpha}$ indices. The explicit expression for

$T^{((\underline{\alpha}_1 \cdots \underline{\alpha}_7))[\underline{\delta}_1 \cdots \underline{\delta}_9]}$ in terms of γ-matrices is a bit more complicated than in d=10, however, it can be defined indirectly through the formula

$$(\lambda \gamma^{\underline{m}_1} \theta) ... (\lambda \gamma^{\underline{m}_9} \theta) = T^{((\underline{\alpha}_1 \cdots \underline{\alpha}_7))[\underline{\delta}_1 \cdots \underline{\delta}_9]} \lambda_{\underline{\alpha}_1} ... \lambda_{\underline{\alpha}_7} \theta_{\underline{\delta}_1} ... \theta_{\underline{\delta}_9} \ (\lambda \gamma^{\underline{m}_1 \cdots \underline{m}_9} \lambda) \tag{1.8}$$

for any $[\underline{m}_1 ... \underline{m}_9]$.

The d=10 and d=11 supersymmetric invariants of (1.2), (1.5) and (1.7) were originally constructed by looking for elements of top ghost number in the pure spinor BRST cohomology [1][2]. Using the N=1 d=10 nilpotent BRST operator $Q_{N=1} = \lambda^\alpha D_\alpha$, the top element in the BRST cohomology is

$$(\lambda \gamma^m \theta)(\lambda \gamma^n \theta)(\lambda \gamma^p \theta)(\theta \gamma_{mnp} \theta). \tag{1.9}$$

Since (1.9) cannot be written as the supersymmetric variation of a BRST-closed operator, and since (1.2) selects out the component of $\lambda^\alpha \lambda^\beta \lambda^\gamma f_{\alpha\beta\gamma}$ proportional to (1.9), (1.2) is supersymmetric if $\lambda^\alpha \lambda^\beta \lambda^\gamma f_{\alpha\beta\gamma}$ is BRST closed, i.e. if $f_{\alpha\beta\gamma}$ satisfies (1.1). Similarly, the top element in the cohomology of the N=2 d=10 BRST operator $Q_{N=2} = \lambda^\alpha D_\alpha + \widehat{\lambda}^{\widehat{\alpha}} D_{\widehat{\alpha}}$ is

$$(\lambda \gamma^{m_1} \theta)(\lambda \gamma^{m_2} \theta)(\lambda \gamma^{m_3} \theta)(\theta \gamma_{m_1 m_2 m_3} \theta) \ (\widehat{\lambda} \gamma^{n_1} \widehat{\theta})(\widehat{\lambda} \gamma^{n_2} \widehat{\theta})(\widehat{\lambda} \gamma^{n_3} \widehat{\theta})(\widehat{\theta} \gamma_{n_1 n_2 n_3} \widehat{\theta}), \tag{1.10}$$

and the top element in the cohomology of the d=11 BRST operator $Q_{d=11} = \lambda^{\underline{\alpha}} D_{\underline{\alpha}}$ is

$$\lambda_{\underline{\alpha}_1} ... \lambda_{\underline{\alpha}_7} T^{((\underline{\alpha}_1 \cdots \underline{\alpha}_7))[\underline{\delta}_1 \cdots \underline{\delta}_9]} \theta_{\underline{\delta}_1} ... \theta_{\underline{\delta}_9}. \tag{1.11}$$

So (1.5) and (1.7) are supersymmetric if

$$Q_{N=2}(\lambda^\alpha \lambda^\beta \lambda^\gamma \widehat{\lambda}^{\widehat{\alpha}} \widehat{\lambda}^{\widehat{\beta}} \widehat{\lambda}^{\widehat{\gamma}} f_{\alpha\beta\gamma \ \widehat{\alpha}\widehat{\beta}\widehat{\gamma}}) = 0 \quad \text{and} \quad Q_{d=11}(\lambda^{\underline{\alpha}_1} ... \lambda^{\underline{\alpha}_7} f_{\underline{\alpha}_1 \cdots \underline{\alpha}_7}) = 0, \tag{1.12}$$

i.e. if (1.4) and (1.6) are satisfied.

Although this pure spinor construction is hard to understand using the conventional method for constructing supersymmetric invariants, it will be easy to explain this construction using the superform (or "ectoplasm") method developed by Gates and collaborators [3][4] for constructing supersymmetric invariants. The superform (or "ectoplasm") method will also be useful for generalizing these d=10 and d=11 invariants in a curved supergravity background.

When constructed using the superform method, the invariants of (1.2), (1.5) and (1.7) will turn out to be natural d=10 and d=11 generalizations of chiral superspace integrals in four dimensions. This is not surprising since, as was shown in [5], there exists a four-dimensional version of the pure spinor formalism whose scattering amplitudes compute chiral F-terms in the d=4 effective action.

In section 2 of this paper, the superform method for constructing N=1 and N=2 supersymmetric invariants in four dimensions will be reviewed. And in section 3, the superform method will be used to construct the N=1 d=10, N=2A d=10, and d=11 supersymmetric invariants of (1.2), (1.5) and (1.7). Surprisingly, the N=2B d=10 supersymmetric invariant of (1.5) does not have an obvious construction using the superform method.

2. Review of Superform (or "Ectoplasm") Method

The superform (or "ectoplasm") method was developed in papers by Gates [3] and by Gates, Grisaru, Knutt-Wehlau and Siegel [4], and has connections with work on "rheonomy" [6] and brane embeddings [7]. The superform method has previously been used to reproduce supergravity actions [3][4], to construct new supersymmetric invariants in three [8], four [9] and six [10] dimensions, and to construct supersymmetric Chern-Simons terms in any dimension [11]. The relation between the superform method and the pure spinor constructions has some similarities with the superaction formalism of [12] and with the relation found by Cederwall, Nilsson and Tsimpis [13] between maximally supersymmetric deformations and spinorial cohomology.

The basic idea of the superform method is to look for a closed superform $J_{M_1\dots M_d}(x,\theta)$ where d is the dimension of spacetime and $M=(m,\mu)$ is either a spacetime vector index m or a spacetime spinor index μ. Note that superforms are graded-antisymmetric, i.e. they are antisymmetric in the vector indices and symmetric in the spinor indices. In terms of $J_{M_1\dots M_d}(x,\theta)$, the supersymmetric invariant is given simply by

$$I=\frac{1}{d!}\epsilon^{m_1\dots m_d}\int d^dx\ J_{m_1\dots m_d}(x,\theta=0). \tag{2.1}$$

When $J_{M_1\dots M_d}$ is closed (i.e. $\partial_{[N}J_{M_1\dots M_d)}=0$ where [ ) denotes commutator for vector indices and anticommutator for spinor indices), I is supersymmetric since

$$\int d^dx\ \frac{\partial}{\partial\theta^\mu}J_{m_1\dots m_d}(x,\theta)=\frac{(-1)^{d+1}}{(d-1)!}\int d^dx\ \partial_{[m_1}J_{m_2\dots m_d]\mu}(x,\theta)=0 \tag{2.2}$$

if one ignores surface terms.

Furthermore, this method is easily generalized to a curved supergravity background by defining

$$I = \frac{1}{d!}\epsilon^{m_1 ... m_d} \int d^d x \; e_{m_d}^{A_d}(x) ... e_{m_1}^{A_1}(x) \; J_{A_1 ... A_d}(x, \theta = 0) \tag{2.3}$$

where $A = (a, \alpha)$ are tangent-superspace indices, $e_m^a(x)$ and $e_m^\alpha(x)$ are the vielbein and gravitino, $J_{A_1 ... A_d}(x, \theta)$ is a covariantly closed d-superform satisfying

$$D_{[B} J_{A_1 ... A_d)} = \frac{d}{2} \; T_{[BA_1|}{}^C \; J_{C|A_2 ... A_d)}, \tag{2.4}$$

and $T_{AB}{}^C$ is the supertorsion. The formula of (2.4) can be derived from the relation

$$E^{A_d} ... E^{A_1} J_{A_1 ... A_d} = dZ^{M_d} ... dZ^{M_1} J_{M_1 ... M_d} \tag{2.5}$$

where $E^A = dZ^M E_M^A$ is the vielbein superform, E_M^A is the supervielbein, $dZ^M = (dx^m, d\theta^\mu)$, and $\partial_{[N} J_{M_1 ... M_d)} = 0$. Note that I of (2.3) is invariant under the gauge transformation

$$\delta J_{A_1 ... A_d} = \frac{1}{(d-1)!} D_{[A_1} \Lambda_{A_2 ... A_d)} - \frac{1}{2(d-2)!} T_{[A_1 A_2|}{}^C \Lambda_{C|A_3 ... A_d)} \tag{2.6}$$

since under (2.6), $\delta J_{m_1 ... m_d} = \frac{1}{(d-1)!} \partial_{[m_1} \Lambda_{m_2 ... m_d)}$.

Solving (2.4) for $J_{A_1 ... A_d}$ only requires knowledge of the supertorsion and supercurvature, so the explicit superfield for the supervielbein is unnecessary for constructing the supersymmetric invariant of (2.3). Since the supervielbein is usually a complicated superfield, this is a big advantage over the conventional approach to constructing supersymmetric invariants in a curved background.

In looking for solutions to (2.4) in a flat background where the only non-zero torsion is $T_{\alpha\beta}{}^c = \gamma_{\alpha\beta}^c$, it will turn out that $J_{a_1 ... a_d}(x, \theta)$ with all vector indices can be related to $J_{a_1 ... a_{d-N} \beta_1 ... \beta_N}(x, \theta)$ with $d - N$ vector indices by acting with N spinor derivatives, i.e.

$$J_{a_1 ... a_d}(x, \theta) = D_{\gamma_1} ... D_{\gamma_N} J_{a_1 ... a_{d-N} \beta_1 ... \beta_N}(x, \theta) \tag{2.7}$$

where the index contractions on the right-hand side of (2.7) need to be worked out. Furthermore, one finds that when N is larger than some fixed value L, $J_{a_1 ... a_{d-N} \beta_1 ... \beta_N}(x, \theta) = 0$. So in a flat background, the supersymmetric invariant can be written as

$$I = \frac{1}{d!}\epsilon^{a_1 ... a_d} \int d^d x \; J_{a_1 ... a_d}(x, \theta = 0) = \int d^d x \; D_{\gamma_1} ... D_{\gamma_L} J_{a_1 ... a_{d-L} \beta_1 ... \beta_L}(x, \theta = 0) \tag{2.8}$$

$$= \int d^d x \int (d^L \theta)_{\gamma_1 ... \gamma_L} \; J_{a_1 ... a_{d-L} \beta_1 ... \beta_L}(x, \theta)$$

for some contraction of the spinor and vector indices. Determining the conditions for $J_{a_1 ... a_{d-L} \beta_1 ... \beta_L}(x, \theta)$ to satisfy (2.4) is equivalent in the conventional approach to finding the appropriate set of constraints for the superfields which allow integration over L θ's.

2.1. N=1 d=4 invariants

To reproduce the standard N=1 d=4 chiral superspace integral using the superform method, one imposes that the maximum number of spinor indices on $J_{A_1 \ldots A_4}(x,\theta)$ is two and that [14]

$$J_{ab\gamma\delta}(x,\theta) = (\gamma_{ab})_{\gamma\delta}\overline{V}(x,\theta), \quad J_{ab\dot\gamma\dot\delta}(x,\theta) = (\gamma_{ab})_{\dot\gamma\dot\delta}V(x,\theta), \tag{2.9}$$

where $a = 0$ to 3 are vector indices, $\alpha = 1$ to 2 and $\dot\alpha = 1$ to 2 are Weyl and anti-Weyl spinor indices, V and $\overline{V}$ are chiral and antichiral superfields satisfying $D_{\dot\gamma}V = 0$ and $D_\gamma \overline{V} = 0$, and $(\gamma_{ab})_{\gamma\delta}$ and $(\gamma_{ab})_{\dot\gamma\dot\delta}$ are the self-dual and anti-self-dual two-form γ-matrices.

In a flat background, the only non-zero torsion is $T_{\alpha\dot\beta}{}^c = \sigma^c_{\alpha\dot\beta}$ and the chirality conditions on V and $\overline{V}$ come from the constraints that $D_{(\alpha}J_{\beta\gamma)ab} = 0$ and $D_{(\dot\alpha}J_{\dot\beta\dot\gamma)ab} = 0$. Furthermore, the gauge parameter Λ_{abc} of (2.6) can be used to gauge $J_{ab\alpha\dot\beta} = 0$. The constraints $D_{\dot\alpha}J_{ab\beta\gamma} = T_{\dot\alpha(\beta}{}^c J_{\gamma)abc}$ and $D_\alpha J_{ab\dot\beta\dot\gamma} = T_{\alpha(\dot\beta}{}^c J_{\dot\gamma)abc}$ imply that $J_{abc\gamma} = \epsilon_{abcd}\sigma^d_{\gamma\dot\beta}D^{\dot\beta}\overline{V}$ and $J_{abc\dot\gamma} = \epsilon_{abcd}\sigma^d_{\beta\dot\gamma}D^\beta V$. And the constraint $D_{(\dot\alpha}J_{\alpha)abc} = T_{\alpha\dot\alpha}{}^d J_{dabc}$ implies that $J_{abcd} = \epsilon_{abcd}(D_\alpha D^\alpha V + D_{\dot\alpha}D^{\dot\alpha}\overline{V})$. So the supersymmetric invariant is

$$I = \frac{1}{4!}\epsilon^{abcd}\int d^4x J_{abcd} = \int d^4x (D_\alpha D^\alpha V + D_{\dot\alpha}D^{\dot\alpha}\overline{V}), \tag{2.10}$$

which reproduces the standard d=4 chiral superspace integral $I = \int d^4x(\int d^2\theta \; V + \int d^2\overline{\theta} \; \overline{V})$.

2.2. N=2 d=4 invariants

For the N=2 d=4 case, one finds that the maximum number of spinor indices on $J_{A_1\ldots A_4}(x,\theta,\widehat\theta)$ is four, and that [9]

$$J_{\alpha\beta\widehat\gamma\widehat\delta}(x,\theta,\widehat\theta) = (\gamma_{ab})_{\alpha\beta}(\gamma^{ab})_{\widehat\gamma\widehat\delta}\overline{W}(x,\theta,\widehat\theta), \quad J_{\dot\alpha\dot\beta\widehat{\dot\gamma}\widehat{\dot\delta}}(x,\theta,\widehat\theta) = (\gamma_{ab})_{\dot\alpha\dot\beta}(\gamma^{ab})_{\widehat{\dot\gamma}\widehat{\dot\delta}}W(x,\theta,\widehat\theta), \tag{2.11}$$

where $\alpha, \dot\beta, \widehat\gamma, \widehat{\dot\delta} = 1$ to 2, W and $\overline{W}$ are chiral and antichiral superfields satisfying the constraints $D_{\dot\gamma}W = D_{\widehat{\dot\delta}}W = 0$ and $D_\gamma\overline{W} = D_{\widehat\delta}\overline{W} = 0$, and all other components of $J_{A_1\ldots A_4}$ with four spinor indices are zero.

In a flat background, the only non-zero torsions are $T_{\alpha\dot\beta}{}^c = \sigma^c_{\alpha\dot\beta}$ and $T_{\widehat\alpha\widehat{\dot\beta}}{}^c = \sigma^c_{\widehat\alpha\widehat{\dot\beta}}$, and the chirality conditions on W and $\overline{W}$ come from the constraints that

$$D_{(\alpha}J_{\beta\gamma)\widehat\beta\widehat\gamma} = D_{(\widehat\alpha}J_{\widehat\beta\widehat\gamma)\beta\gamma} = D_{(\dot\alpha}J_{\dot\beta\dot\gamma)\widehat{\dot\beta}\widehat{\dot\gamma}} = D_{(\widehat{\dot\alpha}}J_{\widehat{\dot\beta}\widehat{\dot\gamma})\dot\beta\dot\gamma} = 0. \tag{2.12}$$

As in the N=1 d=4 case, the vector components of $J_{A_1\ldots A_4}$ can be determined from the spinor components of (2.11) using the constraints of (2.4). One finds that $J_{abcd} = \epsilon_{abcd}(D_\alpha D^\alpha D_{\widehat\beta} D^{\widehat\beta} W + D_{\dot\alpha} D^{\dot\alpha} D_{\widehat{\dot\beta}} D^{\widehat{\dot\beta}} \overline{W})$, so the supersymmetric invariant

$$I = \frac{1}{4!}\epsilon^{abcd}\int d^4x J_{abcd} = \int d^4x (D_\alpha D^\alpha D_{\widehat\beta} D^{\widehat\beta} W + D_{\dot\alpha} D^{\dot\alpha} D_{\widehat{\dot\beta}} D^{\widehat{\dot\beta}} \overline{W}) \tag{2.13}$$

coincides with the standard N=2 chiral superspace integral $I = \int d^4x (\int d^2\theta d^2\widehat\theta\ W + \int d^2\overline\theta \int d^2\widehat{\overline\theta}\ \overline{W})$.

3. Superform Method in Higher Dimensions

3.1. N=1 d=10 invariants

In any even spacetime dimension $d = 2R$, there is a natural generalization of the N=1 d=4 formula of (2.9) for the superforms. The generalization is that the maximum number of spinor indices of $J_{A_1\ldots A_d}(x,\theta)$ is $R = \frac{d}{2}$ and that

$$J_{a_1\ldots a_R \beta_1\ldots\beta_R}(x,\theta) = (\gamma_{a_1\ldots a_R})_{(\beta_1\beta_2}\ \ f_{\beta_3\ldots\beta_R)}(x,\theta) \tag{3.1}$$

where $f_{\alpha_1\ldots\alpha_{R-2}}(x,\theta)$ is a superfield satisfying the constraint

$$\lambda^\beta \lambda^{\alpha_1} \ldots \lambda^{\alpha_{R-2}} D_\beta f_{\alpha_1\ldots\alpha_{R-2}} = 0, \tag{3.2}$$

$(\gamma_{a_1\ldots a_R})_{\beta\gamma} = (\gamma_{a_1\ldots a_R})_{\gamma\beta}$ is the self-dual $\frac{d}{2}$-form γ-matrix, and λ^α is a bosonic spinor satisfying the condition that $\lambda\gamma^c\lambda = 0$ for $c = 0$ to $d-1$.

To show that (3.1) satisfies (2.4) in a flat background where the only non-vanishing torsion is $T_{\alpha\beta}{}^c = \gamma^c_{\alpha\beta}$, note that (3.2) implies that $\lambda^\gamma\lambda^{\beta_1}\ldots\lambda^{\beta_R} D_\gamma J_{\beta_1\ldots\beta_R a_1\ldots a_R} = 0$. Since $\lambda\gamma^c\lambda = 0$, this implies that

$$D_{(\gamma} J_{\beta_1\ldots\beta_R) a_1\ldots a_R} = \gamma^c_{(\gamma\beta_1} K_{\beta_2\ldots\beta_R) a_1\ldots a_R c} \tag{3.3}$$

for some $K_{\beta_2\ldots\beta_R a_1\ldots a_R c}$. If one chooses $J_{\beta_1\ldots\beta_{R-1} a_1\ldots a_{R+1}}$ such that it is proportional to $K_{\beta_1\ldots\beta_{R-1} a_1\ldots a_{R+1}}$, the first non-trivial constraint of (2.4) is satisfied. Furthermore, the gauge invariance of (2.6) implies that $J_{a_1\ldots a_R \beta_1\ldots\beta_R}$ is defined up to the gauge transformation

$$\delta J_{a_1\ldots a_R \beta_1\ldots\beta_R} = \frac{1}{(R-1)!} D_{(\beta_1}\Lambda_{\beta_2\ldots\beta_R) a_1\ldots a_R} - \frac{1}{2(R-2)!}\gamma^c_{(\beta_1\beta_2}\Lambda_{\beta_3\ldots\beta_R) a_1\ldots a_R c}. \tag{3.4}$$

As in the d=4 case, components of $J_{A_1...A_d}$ with more than $\frac{d}{2}$ vector components can be constructed from spinor derivatives of $J_{a_1...a_R\beta_1...\beta_R}$ of (3.1) by using the constraints of (2.4). In a flat background, the supersymmetric invariant will therefore have the form

$$I = \frac{1}{d!}\epsilon^{a_1...a_d}\int d^dx\ J_{a_1...a_d}(x,\theta=0) = \int d^dx \int (d^R\theta)_{\delta_1...\delta_R}\ f_{\beta_1...\beta_{R-2}}(x,\theta) \qquad (3.5)$$

where the index contractions need to be worked out.

When $d=10$, $J_{a_1...a_5\beta_1...\beta_5}(x,\theta) = (\gamma_{a_1...a_5})_{(\beta_1\beta_2}\ f_{\beta_3\beta_4\beta_5)}(x,\theta)$ where $f_{\alpha\beta\gamma}$ satisfies the same constraints as in (1.1). To show that (3.5) reproduces the supersymmetric invariant of (1.2), note that the gauge invariance of (3.4) implies that (3.5) is invariant under

$$\delta f_{\alpha\beta\gamma} = \frac{1}{2}D_{(\alpha}\Sigma_{\beta\gamma)} + \frac{1}{2}\gamma^c_{(\alpha\beta}\Omega_{\gamma)c} \qquad (3.6)$$

where

$$\Lambda^{a_1...a_5}_{\beta_1\beta_2\beta_3\beta_4} = \frac{1}{4}(\gamma^{a_1...a_5})_{(\beta_1\beta_2}\Sigma_{\beta_3\beta_4)}, \quad \Lambda^{a_1...a_5c}_{\beta_1\beta_2\beta_3} = \frac{1}{240}(\gamma^{[a_1...a_5})_{(\beta_1\beta_2}\Omega^{c]}_{\beta_3)}. \qquad (3.7)$$

In relating (3.4) and (3.6), one needs to use the d=10 identity $(\gamma_c)_{(\beta_1\beta_2}(\gamma^{ca_1a_2a_3a_4})_{\beta_3\beta_4)} = 0$.

The gauge invariance of (3.6) implies that (3.5) only depends on the γ-matrix traceless part of $f_{\alpha\beta\gamma}$ and is invariant under

$$\delta(\lambda^\alpha\lambda^\beta\lambda^\gamma f_{\alpha\beta\gamma}) = \lambda^\alpha D_\alpha(\lambda^\beta\lambda^\gamma\Sigma_{\beta\gamma}) = Q_{N=1}(\lambda^\beta\lambda^\gamma\Sigma_{\beta\gamma}). \qquad (3.8)$$

So (3.5) is independent of BRST-trivial deformations of $\lambda^\alpha\lambda^\beta\lambda^\gamma f_{\alpha\beta\gamma}$. Since the unique state with three λ's in the BRST cohomology is (1.9), (3.5) selects out the component of $\lambda^\alpha\lambda^\beta\lambda^\gamma f_{\alpha\beta\gamma}$ proportional to (1.9), and is therefore proportional to

$$T^{((\beta_1\beta_2\beta_3))[\delta_1...\delta_5]}\int d^{10}x\int(d^5\theta)_{\delta_1...\delta_5}\ f_{\beta_1\beta_2\beta_3}(x,\theta) \qquad (3.9)$$

of (1.2).

To generalize this N=1 d=10 supersymmetric invariant in a curved supergravity background, one first defines $J_{a_1...a_5\beta_1...\beta_5}(x,\theta) = (\gamma_{a_1...a_5})_{(\beta_1\beta_2}f_{\beta_3...\beta_5)}(x,\theta)$ as in (3.1), but where D_β of (3.2) is now the spinor derivative in a curved background. One then needs to compute the other components of $J_{A_1...A_{10}}$ in terms of $f_{\alpha\beta\gamma}(x,\theta)$ using the constraints of (2.4). Finally, one plugs the $\theta=0$ components of $J_{A_1...A_{10}}$ into the supersymmetric invariant

$$I = \frac{1}{10!}\epsilon^{m_1...m_{10}}\int d^{10}x\ e^{A_{10}}_{m_{10}}(x)...e^{A_1}_{m_1}(x)\ J_{A_1...A_{10}}(x,\theta=0) \qquad (3.10)$$

where $e^a_m(x)$ and $e^\alpha_m(x)$ are the ten-dimensional vielbein and gravitino.

3.2. N=2A d=10 invariants

In any even spacetime dimension $d = 2R$, a natural generalization of the N=2 d=4 formula of (2.11) is that the maximum number of spinor indices of $J_{A_1 \ldots A_d}(x,\theta,\widehat{\theta})$ is $d = 2R$ and that

$$J_{\alpha_1\ldots\alpha_R\widehat{\beta}_1\ldots\widehat{\beta}_R}(x,\theta,\widehat{\theta}) = (\gamma_{c_1\ldots c_R})_{(\alpha_1\alpha_2} \quad f_{\alpha_3\ldots\alpha_R)(\widehat{\beta}_1\ldots\widehat{\beta}_{R-2}}(x,\theta,\widehat{\theta}) \quad (\gamma^{c_1\ldots c_R})_{\widehat{\beta}_{R-1}\widehat{\beta}_R)} \tag{3.11}$$

where $f_{\alpha_1\ldots\alpha_{R-2}\widehat{\beta}_1\ldots\widehat{\beta}_{R-2}}(x,\theta,\widehat{\theta})$ is a superfield satisfying the constraints

$$\lambda^\gamma\lambda^{\alpha_1}\ldots\lambda^{\alpha_{R-2}}\widehat{\lambda}^{\widehat{\beta}_1}\ldots\widehat{\lambda}^{\widehat{\beta}_{R-2}}D_\gamma f_{\alpha_1\ldots\alpha_{R-2}\widehat{\beta}_1\ldots\widehat{\beta}_{R-2}} = 0, \tag{3.12}$$

$$\widehat{\lambda}^{\widehat{\gamma}}\widehat{\lambda}^{\widehat{\beta}_1}\ldots\widehat{\lambda}^{\widehat{\beta}_{R-2}}\lambda^{\alpha_1}\ldots\lambda^{\alpha_{R-2}}D_{\widehat{\gamma}} f_{\alpha_1\ldots\alpha_{R-2}\widehat{\beta}_1\ldots\widehat{\beta}_{R-2}} = 0,$$

and λ^α and $\widehat{\lambda}^{\widehat{\beta}}$ are bosonic spinors satisfying the conditions that $\lambda\gamma^c\lambda = \widehat{\lambda}\gamma^c\widehat{\lambda} = 0$ for $c = 0$ to $d-1$.

To show that (3.11) satisfies (2.4) in a flat background where the only non-vanishing torsions are $T_{\alpha\beta}{}^c = \gamma^c_{\alpha\beta}$ and $T_{\widehat{\alpha}\widehat{\beta}}{}^c = \gamma^c_{\widehat{\alpha}\widehat{\beta}}$, note that (3.12) implies that

$$\lambda^\gamma\lambda^{\alpha_1}\ldots\lambda^{\alpha_R}\widehat{\lambda}^{\widehat{\beta}_1}\ldots\widehat{\lambda}^{\widehat{\beta}_R}D_\gamma J_{\alpha_1\ldots\alpha_R\widehat{\beta}_1\ldots\widehat{\beta}_R} = 0, \tag{3.13}$$

$$\widehat{\lambda}^{\widehat{\gamma}}\widehat{\lambda}^{\widehat{\beta}_1}\ldots\widehat{\lambda}^{\widehat{\beta}_R}\lambda^{\alpha_1}\ldots\lambda^{\alpha_R}D_{\widehat{\gamma}} J_{\alpha_1\ldots\alpha_R\widehat{\beta}_1\ldots\widehat{\beta}_R} = 0.$$

Since $\lambda\gamma^c\lambda = \widehat{\lambda}\gamma^c\widehat{\lambda} = 0$, this implies that

$$D_{(\gamma}J_{\alpha_1\ldots\alpha_R)\widehat{\beta}_1\ldots\widehat{\beta}_R} = \gamma^c_{(\gamma\alpha_1}K_{\alpha_2\ldots\alpha_R)c\widehat{\beta}_1\ldots\widehat{\beta}_R} + \gamma^c_{(\widehat{\beta}_1\widehat{\beta}_2}K_{\widehat{\beta}_3\ldots\widehat{\beta}_R)c\gamma\alpha_1\ldots\alpha_R}, \tag{3.14}$$

$$D_{(\widehat{\gamma}}J_{\widehat{\beta}_1\ldots\widehat{\beta}_R)\alpha_1\ldots\alpha_R} = \gamma^c_{(\widehat{\gamma}\widehat{\beta}_1}K_{\widehat{\beta}_2\ldots\widehat{\beta}_R)c\alpha_1\ldots\alpha_R} + \gamma^c_{(\alpha_1\alpha_2}K_{\alpha_3\ldots\alpha_R)c\widehat{\gamma}\widehat{\beta}_1\ldots\widehat{\beta}_R},$$

for some choice of K's with one vector index and $d-1$ spinor indices. If one sets J's with one vector index and $d-1$ spinor indices to be proportional to these K's, the first non-trivial condition coming from (2.4) is satisfied. Furthermore, the gauge invariance of (2.6) implies that $f_{\alpha_1\ldots\alpha_{R-2}\widehat{\beta}_1\ldots\widehat{\beta}_{R-2}}$ is defined up to the gauge transformation

$$\delta f_{\alpha_1\ldots\alpha_{R-2}\widehat{\beta}_1\ldots\widehat{\beta}_{R-2}} = D_{(\alpha_1}\Sigma_{\alpha_2\ldots\alpha_{R-2})\widehat{\beta}_1\ldots\widehat{\beta}_{R-2}} + D_{(\widehat{\beta}_1}\widehat{\Sigma}_{\widehat{\beta}_2\ldots\widehat{\beta}_{R-2})\alpha_1\ldots\alpha_{R-2}} \tag{3.15}$$

$$+\gamma^c_{(\alpha_1\alpha_2}\Omega_{\alpha_3\ldots\alpha_{R-2})\widehat{\beta}_1\ldots\widehat{\beta}_{R-2}c} + \gamma^c_{(\widehat{\beta}_1\widehat{\beta}_2}\widehat{\Omega}_{\widehat{\beta}_3\ldots\widehat{\beta}_{R-2})\alpha_1\ldots\alpha_{R-2}c}.$$

As in the N=2 d=4 case, components of $J_{A_1\dots A_d}$ with vector components can be constructed from spinor derivatives of $J_{\alpha_1\dots\alpha_R\widehat\beta_1\dots\widehat\beta_R}$ of (3.11) by using the constraints of (2.4). In a flat background, the supersymmetric invariant will therefore have the form

$$I = \epsilon^{a_1\dots a_d}\int d^d x\ J_{a_1\dots a_d}(x,\theta=\widehat\theta=0) \tag{3.16}$$

$$=\int d^d x\int (d^R\theta)_{\gamma_1\dots\gamma_R}(d^R\widehat\theta)_{\widehat\delta_1\dots\widehat\delta_R}\ f_{\alpha_1\dots\alpha_{R-2}\widehat\beta_1\dots\widehat\beta_{R-2}}$$

where the index contractions need to be worked out.

When $d=10$,

$$J_{\alpha_1\dots\alpha_5\widehat\beta_1\dots\widehat\beta_5}(x,\theta,\widehat\theta) = (\gamma_{c_1\dots c_5})_{(\alpha_1\alpha_2}\ \ f_{\alpha_3\alpha_4\alpha_5)(\widehat\beta_1\widehat\beta_2\widehat\beta_3}(x,\theta,\widehat\theta)\ \ (\gamma^{c_1\dots c_5})_{\widehat\beta_4\widehat\beta_5)} \tag{3.17}$$

where $f_{\alpha_1\alpha_2\alpha_3\widehat\beta_1\widehat\beta_2\widehat\beta_3}$ satisfies the same constraints as in (1.4). However, since

$$(\gamma_{c_1\dots c_5})_{\alpha_1\alpha_2}\ (\gamma^{c_1\dots c_5})_{\widehat\beta_1\widehat\beta_2} \tag{3.18}$$

vanishes when α and $\widehat\beta$ are d=10 spinors of the same chirality, (3.17) can only be used for the N=2A case. So there is no obvious way to construct the N=2B supersymmetric invariant of (1.5) using the superform method.

To show that (3.16) reproduces the supersymmetric invariant of (1.5) for the N=2A case, note that (3.15) implies that (3.16) is independent of the γ-matrix traceless components of $f_{\alpha_1\alpha_2\alpha_3\widehat\beta_1\widehat\beta_2\widehat\beta_3}$ and is invariant under BRST-trivial deformations of the form

$$\delta(\lambda^{\alpha_1}\lambda^{\alpha_2}\lambda^{\alpha_3}\widehat\lambda^{\widehat\beta_1}\widehat\lambda^{\widehat\beta_2}\widehat\lambda^{\widehat\beta_3}f_{\alpha_1\alpha_2\alpha_3\widehat\beta_1\widehat\beta_2\widehat\beta_3}) = \tag{3.19}$$

$$Q_{N=2}(\lambda^{\alpha_2}\lambda^{\alpha_3}\widehat\lambda^{\widehat\beta_1}\widehat\lambda^{\widehat\beta_2}\widehat\lambda^{\widehat\beta_3}\Sigma_{\alpha_2\alpha_3\widehat\beta_1\widehat\beta_2\widehat\beta_3}+\lambda^{\alpha_1}\lambda^{\alpha_2}\lambda^{\alpha_3}\widehat\lambda^{\widehat\beta_2}\widehat\lambda^{\widehat\beta_3}\widehat\Sigma_{\alpha_1\alpha_2\alpha_3\widehat\beta_2\widehat\beta_3}).$$

Since the unique state with three λ's and three $\widehat\lambda$'s in the BRST cohomology is (1.10), (3.16) selects out the component of $\lambda^{\alpha_1}\lambda^{\alpha_2}\lambda^{\alpha_3}\widehat\lambda^{\widehat\beta_1}\widehat\lambda^{\widehat\beta_2}\widehat\lambda^{\widehat\beta_3}f_{\alpha_1\alpha_2\alpha_3\widehat\beta_1\widehat\beta_2\widehat\beta_3}$ proportional to (1.10), and therefore reproduces the supersymmetric invariant of (1.5).

3.3. d=11 invariants

Finally, it will be shown how to construct the d=11 supersymmetric invariant of (1.7) using the superform method. Although there is no obvious generalization of the d=4 formulas to odd dimensions, one can construct the d=11 invariant of (1.7) by assuming that the maximum number of spinor indices of $J_{A_1...A_{11}}(x,\theta)$ is $L=9$ and that

$$J_{\underline{c}_1\underline{c}_2\underline{\alpha}_1\cdots\underline{\alpha}_9}(x,\theta) = (\gamma_{\underline{c}_1\underline{c}_2})_{(\underline{\alpha}_1\underline{\alpha}_2}\ \ f_{\underline{\alpha}_3\cdots\underline{\alpha}_9)}(x,\theta) \tag{3.20}$$

where $f_{\underline{\alpha}_1\cdots\underline{\alpha}_9}(x,\theta)$ is a superfield satisfying the constraint of (1.6) that

$$\lambda^{\underline{\gamma}}\lambda^{\underline{\alpha}_1}...\lambda^{\underline{\alpha}_9}D_{\underline{\gamma}}f_{\underline{\alpha}_1\cdots\underline{\alpha}_9} = 0, \tag{3.21}$$

$(\gamma_{\underline{c}_1\underline{c}_2})_{\underline{\beta\gamma}} = (\gamma_{\underline{c}_1\underline{c}_2})_{\underline{\gamma\beta}}$ is the two-form d=11 γ-matrix, and $\lambda^{\underline{\alpha}}$ is a bosonic spinor satisfying the condition that $\lambda\gamma^{\underline{c}}\lambda = 0$ for $\underline{c} = 0$ to 10.

To show that (3.20) satisfies (2.4) in a flat background where the only non-vanishing torsion is $T_{\underline{\alpha\beta}}{}^{\underline{c}} = \gamma^{\underline{c}}_{\underline{\alpha\beta}}$, note that (3.21) implies that $\lambda^{\underline{\gamma}}\lambda^{\underline{\alpha}_1}...\lambda^{\underline{\alpha}_9}D_{\underline{\gamma}}J_{\underline{\alpha}_1\cdots\underline{\alpha}_9\underline{c}_1\underline{c}_2} = 0$. Since $\lambda\gamma^{\underline{c}}\lambda = 0$, this implies that

$$D_{(\underline{\gamma}}J_{\underline{\alpha}_1\cdots\underline{\alpha}_9)\underline{c}_1\underline{c}_2} = \gamma^{\underline{b}}_{(\underline{\gamma\alpha}_1}K_{\underline{\alpha}_2\cdots\underline{\alpha}_9)\underline{c}_1\underline{c}_2\underline{b}} \tag{3.22}$$

for some $K_{\underline{\alpha}_2\cdots\underline{\alpha}_9\underline{c}_1\underline{c}_2\underline{b}}$. If one chooses $J_{\underline{\alpha}_1\cdots\underline{\alpha}_8\underline{c}_1\underline{c}_2\underline{c}_3}$ to be proportional to $K_{\underline{\alpha}_1\cdots\underline{\alpha}_8\underline{c}_1\underline{c}_2\underline{c}_3}$, one finds that the first non-trivial constraint of (2.4) is satisfied. Furthermore, the gauge invariance of (2.6) implies that $f_{\underline{\alpha}_1\cdots\underline{\alpha}_7}$ is defined up to the gauge transformation

$$\delta f_{\underline{\alpha}_1\cdots\underline{\alpha}_7} = D_{(\underline{\alpha}_1}\Sigma_{\underline{\alpha}_2\cdots\underline{\alpha}_7)} + \gamma^{\underline{c}}_{(\underline{\alpha}_1\underline{\alpha}_2}\Omega_{\underline{\alpha}_3\cdots\underline{\alpha}_7)\underline{c}} \tag{3.23}$$

where the $d=11$ γ-matrix identity $(\gamma_c)_{(\beta_1\beta_2}(\gamma^{cd})_{\beta_3\beta_4)} = 0$ has been used.

As before, components of $J_{A_1...A_{11}}$ with more than two vector components can be constructed from spinor derivatives of $J_{\underline{c}_1\underline{c}_2\underline{\alpha}_1\cdots\underline{\alpha}_9}$ of (3.20) by using the constraints of (2.4). In a flat background, the supersymmetric invariant will therefore have the form

$$I = \frac{1}{11!}\epsilon^{\underline{c}_1\cdots\underline{c}_{11}}\int d^{11}x\ J_{\underline{c}_1\cdots\underline{c}_{11}}(x,\theta=0) = \int d^dx\int (d^9\theta)_{\underline{\delta}_1\cdots\underline{\delta}_9}\ f_{\underline{\alpha}_1\cdots\underline{\alpha}_7} \tag{3.24}$$

where the index contractions need to be worked out.

As in the other cases, the gauge invariance of (3.23) implies that (3.24) only depends on the γ-matrix traceless components of $f_{\underline{\alpha}_1\cdot\cdot\underline{\alpha}_7}$ and is invariant under the BRST-trivial deformation

$$\delta(\lambda^{\underline{\alpha}_1}...\lambda^{\underline{\alpha}_7}f_{\underline{\alpha}_1\cdots\underline{\alpha}_7}) = Q_{d=11}(\lambda^{\underline{\alpha}_2}...\lambda^{\underline{\alpha}_7}\Sigma_{\underline{\alpha}_2\cdots\underline{\alpha}_7}). \tag{3.25}$$

Since the unique state with seven λ's in the BRST cohomology is (1.11), (3.24) selects out the component of $\lambda^{\underline{\alpha}_1}...\lambda^{\underline{\alpha}_7} f_{\underline{\alpha}_1\cdots\underline{\alpha}_7}$ proportional to (1.11), and is therefore proportional to

$$T^{((\underline{\alpha}_1\cdots\underline{\alpha}_7))[\underline{\delta}_1\cdots\underline{\delta}_9]}\int d^{11}x\int(d^9\theta)_{\underline{\delta}_1\cdots\underline{\delta}_9}\ f_{\underline{\alpha}_1\cdots\underline{\alpha}_7}(x,\theta) \tag{3.26}$$

of (1.7).

To generalize this $d=11$ supersymmetric invariant in a curved supergravity background, one first defines $J_{\underline{c}_1\underline{c}_2\underline{\alpha}_1\cdots\underline{\alpha}_9}(x,\theta)=(\gamma_{\underline{c}_1\underline{c}_2})_{(\underline{\alpha}_1\underline{\alpha}_2}f_{\underline{\alpha}_3\cdots\underline{\alpha}_9)}(x,\theta)$ as in (3.20) where $D_{\underline{\beta}}$ of (3.21) is now the spinor derivative in a curved background. One then computes the other components of $J_{A_1...A_{11}}$ in terms of $f_{\underline{\alpha}_1\cdots\underline{\alpha}_7}(x,\theta)$ using the constraints of (2.4). Finally, one plugs the $\theta=0$ components of $J_{A_1...A_{11}}$ into the supersymmetric invariant

$$I=\frac{1}{11!}\epsilon^{\underline{m}_1\cdots\underline{m}_{11}}\int d^{11}x\ e^{A_{11}}_{\underline{m}_{11}}(x)...e^{A_1}_{\underline{m}_1}(x)\ J_{A_1...A_{11}}(x,\theta=0) \tag{3.27}$$

where $e^{a}_{\underline{m}}(x)$ and $e^{\alpha}_{\underline{m}}(x)$ are the eleven-dimensional vielbein and gravitino.

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